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TECHNICAL NOTE

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DETERMINING INERTIAS BY USING THE AMPLITUDE DECAY RATE OF A MECHANICAL OSCILLATING SYSTEM

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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DEFINITION OF SYMBOLS

C	System constant ($k/4\pi^2$)
E	System efficiency (efficiency factor)
G	Modulus of rigidity of the torsion rod
I	Mass moment of inertia of the object
I_o	Mass moment of inertia of the attached object about its c.g.
J	Polar moment of inertia of the torsion rod's cross-sectional area
L	Length of the torsion rod
M	Mass of the attached object
N	Total number of cycles used in the "Test Run"*
2N	Number of energy twists in the test run
T	Torque or applied moment
a	Radius of the disk
b	Radius of the torsion rod
f	Tangential force at the periphery of the disk
h	Height (or thickness) of the disk
k	Torsion rod constant (torque stiffness)

*The "Test Run" is that predetermined portion of the curve within the intercepts of the chord line.

DEFINITION OF SYMBOLS (Cont'd)

p	Density of the material
r	Radius of the elemental ring
t	Time of one oscillation
w_1	Angular frequency ($2\pi/t$)
θ	Angle about rotational axis of the torsion rod
θ_1	Amplitude (in radians) at beginning of test run
$\dot{\theta}$	Angular velocity
$\ddot{\theta}$	The resulting angular acceleration (response)
θ_M	Mid-amplitude (point of tangency)
θ_N	Amplitude (in radians) at end of test run
π	3.1415926535

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TECHNICAL NOTE D- 1114

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SUMMARY

A series of tests and experiments have been performed to develop a rapid, accurate,and economical method for measuring mass moments of inertia. A torsional system, consisting of an extruded steel rod, an electronic-counter timer, and a scale for measuring decay rates was employed. Various precision test objects were attached to this system and measured in controlled and uncontrolled environments.

These tests provided a practical means of utilizing amplitude decay rates to determine the average efficiency at which an oscillating system functions. No attempt is made in this report to define or evaluate the specific conditions causing amplitude decay.

Investigation revealed that decay rate or damping is independent of mass and inertia for a specific configuration, but is directly related to some combination of environment and system configuration. This relationship is determined and applied to inertia measuring methods.

INTRODUCTION

In the design of space vehicles, the prediction of vehicle response to applied forces must be sufficiently accurate to assure mission accomplishment. These predicted responses will have validity only if the vehicle's resistance, which depends directly and only on its mass, balance and inertia, is accurately known. This knowledge may be gained analytically, from documentation, on simple regularly shaped configurations. However, incomplete documentation, irregularity of shape and complexity of design rule out this approach in most cases. Hence, in order to reach the desired level of accuracy, the mass characteristics must be measured.

Generalized investigations were conducted on spring, compound, bifilar and torsional pendulum systems. Preliminary results indicated the torsional system to be the most reliable of those tested. The selection of this system may be, in part, attributed to the following: undesirable motions are easier to control; bearings are eliminated; tare weight is reduced; mathematical considerations are simplified; loading, and operation in general, is more easily performed.

It became evident early in the tests that the torsion rod constant varied inversely with the temperature. In a similar manner the time per cycle varied with temperature, surface area and the configuration being measured. Investigations were then directed toward a method based on the mathematical formula $I = Ct^2$, (I = inertia, C = system constant, and t = time for one cycle) to account for these varying influences. A series of tests were run using test pieces having different mass moments of inertia, and curves of amplitudes versus cycles were plotted. These curves proved decay or damping to be independent of inertia. In addition, these curves provided an accurate means of determining the system efficiency.

The efficiency factor was incorporated into the basic equation giving the new formula, $I = ECt^2$. When the correction factor is included, the measured inertias show very close agreement to calculated values of the standard inertia objects.

APPARATUS

Figure 1 illustrates an overall picture of the test facility used. A 1/4-inch-(0.25-inch-)diameter torsion rod 65 inches long (nominal) was the basic frequency generator. This rod was fixed to an A-frame with approximately 9 1/2 feet of clearance. Connected to the lower end of the torsion rod was a 2 × 2 × 2-inch aluminum adapter block which enabled fastening of the test objects in the two required planes. This was the basic "gear."

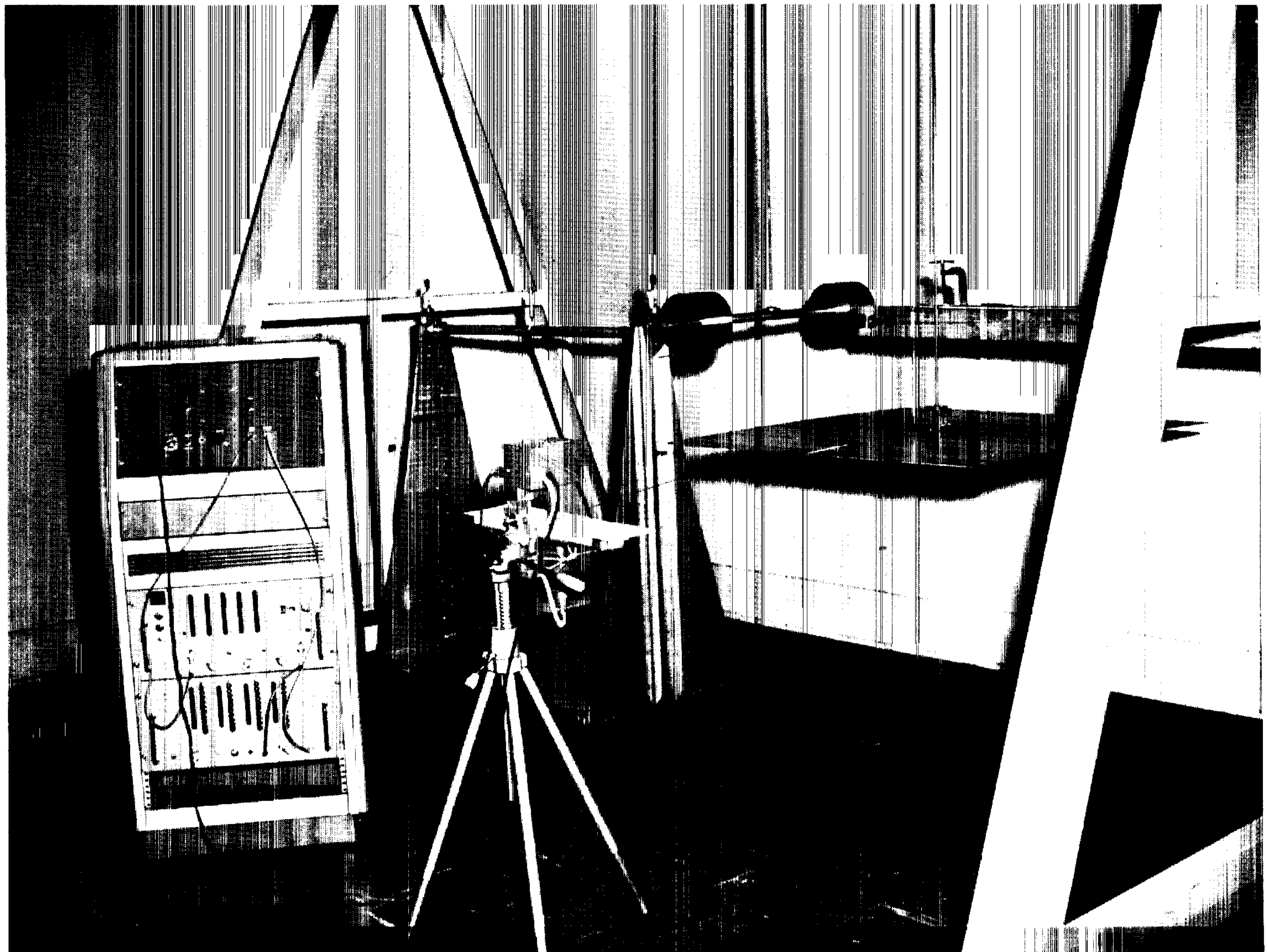

FIGURE 1. OVERALL TEST SETUP

One test object used was a precision-manufactured flat stainless steel plate 0.122 inch thick, 39.484 inches long, and 36.031 inches wide. The weight of this plate was 49.285 pounds.

A photocell and light source was located perpendicular to the axis of rotation. The breaking of the light beam by a 1/16-inch diameter aluminum wand attached to the test object energized the electronic counter-timer.

A Beckman-Berkeley Pendulum Timer was used to count the number of cycles and the time for a given number of cycles.

The total angle of twist that the oscillating system made during each cycle of operation was accurately obtained by visually reading the position of a projected "arrow" on a protractor at the instant the rotating mass changed directions. At this instant, the system has a maximum amount of stored energy.

The protractor used to measure angular displacement was graduated in increments of 0.002 radian and readings were readily estimated to 0.001 radian (FIG. 2).

This protractor was scaled for use on a 140-inch radius, and to read the angle of twist of the rod directly.

An illuminated arrow point was projected onto this radian scaled protractor by a small mirror magnetically mounted on the torsion rod base (FIG. 3 and 4).

Temperature was recorded by a standard laboratory centrigrade thermometer.

This test facility also included four 2.00-pound weights and four 7.97-pound weights for the purpose of adding inertia to the horizontal flate plate as shown in Figure 5.

Two precision-machined "Dumbbells" were used to calibrate the system and verify conclusions.

The one flat stainless steel plate was chosen as a test object because: (1) weight, center of gravity, and mass moments of inertia can be calculated very accurately; (2) object can be tested in such a

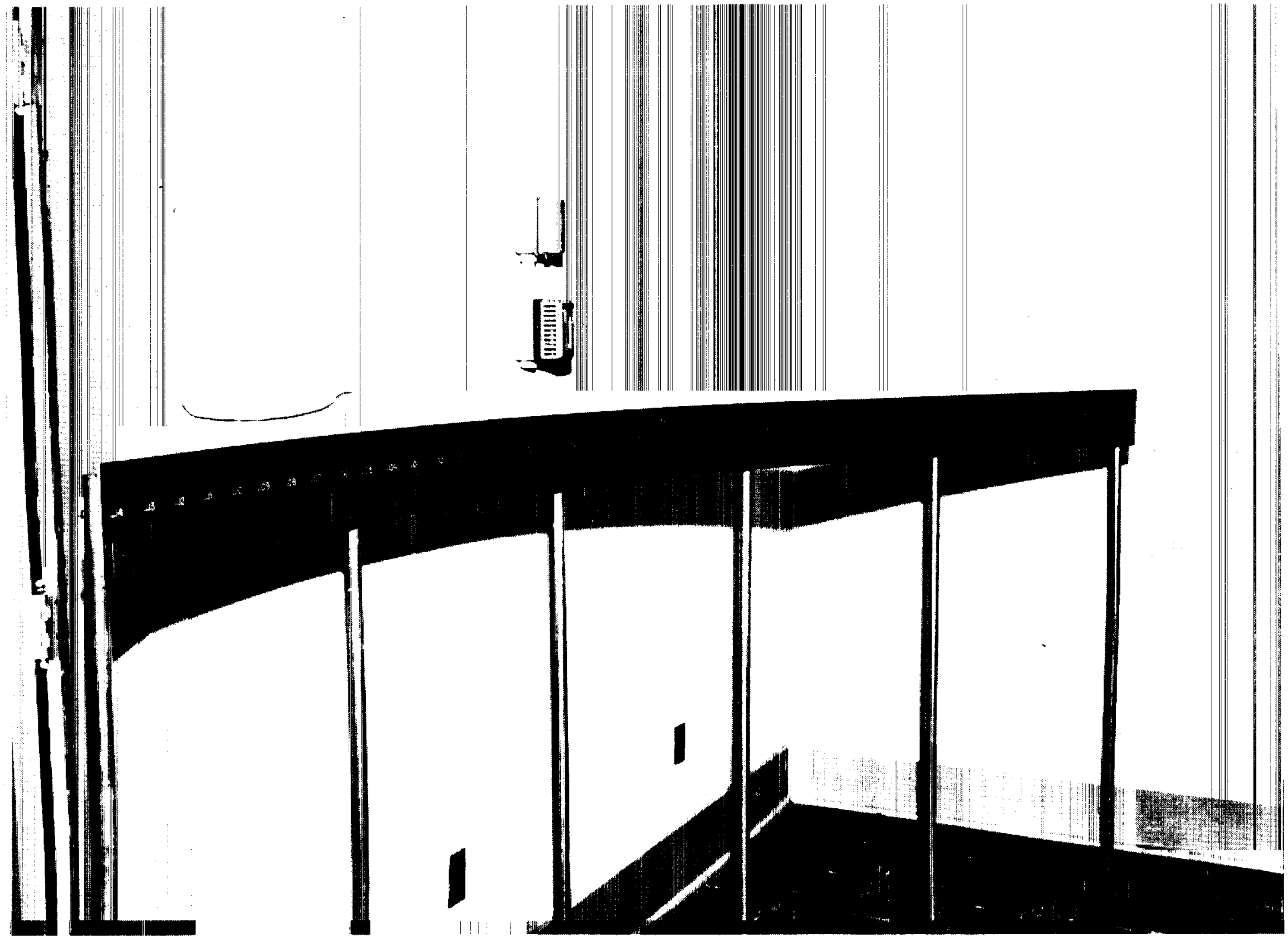

FIGURE 2. RADIAN PROTRACTOR

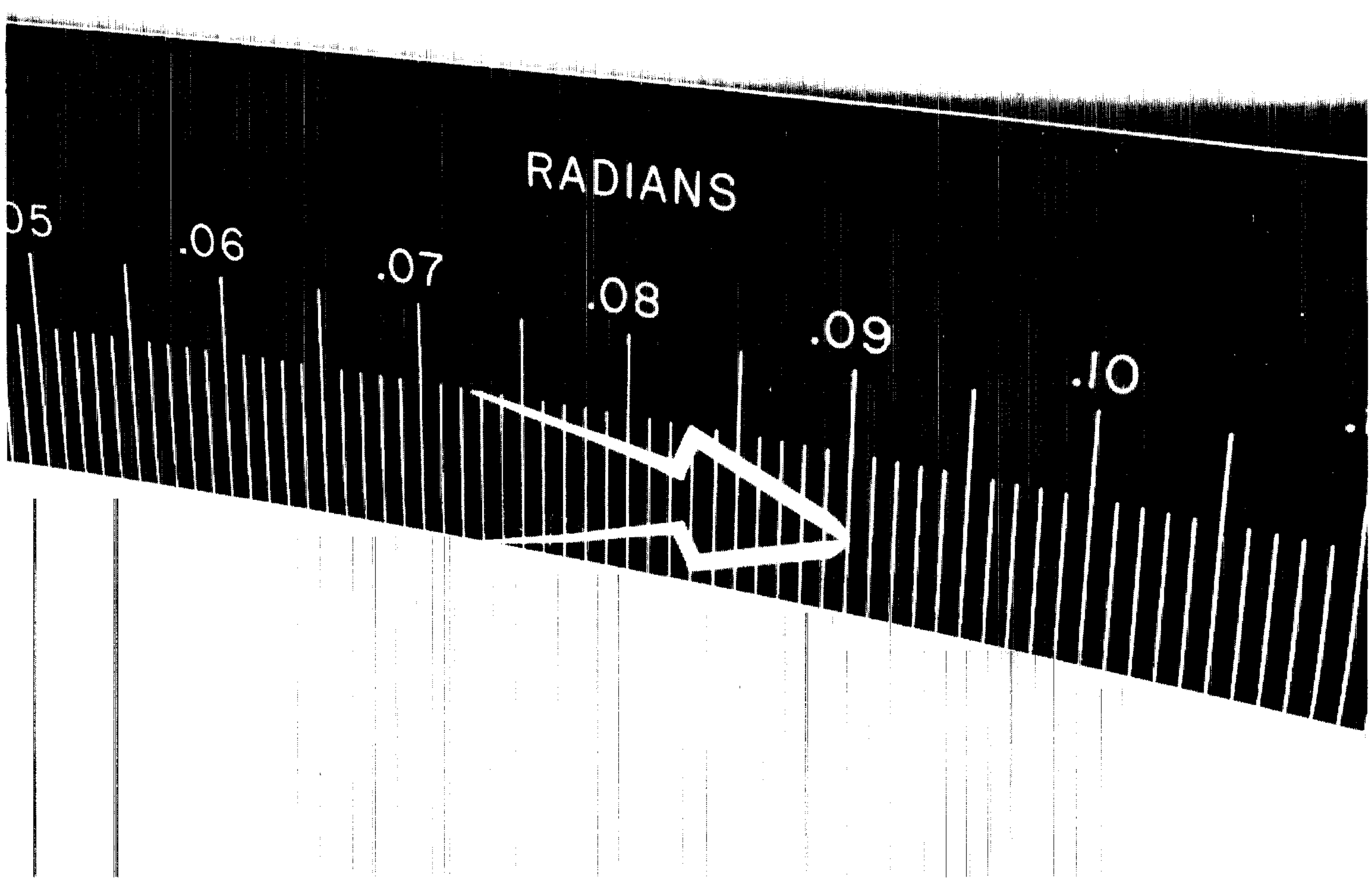

FIGURE 3. ILLUMINATED ARROW ON PROTRACTOR

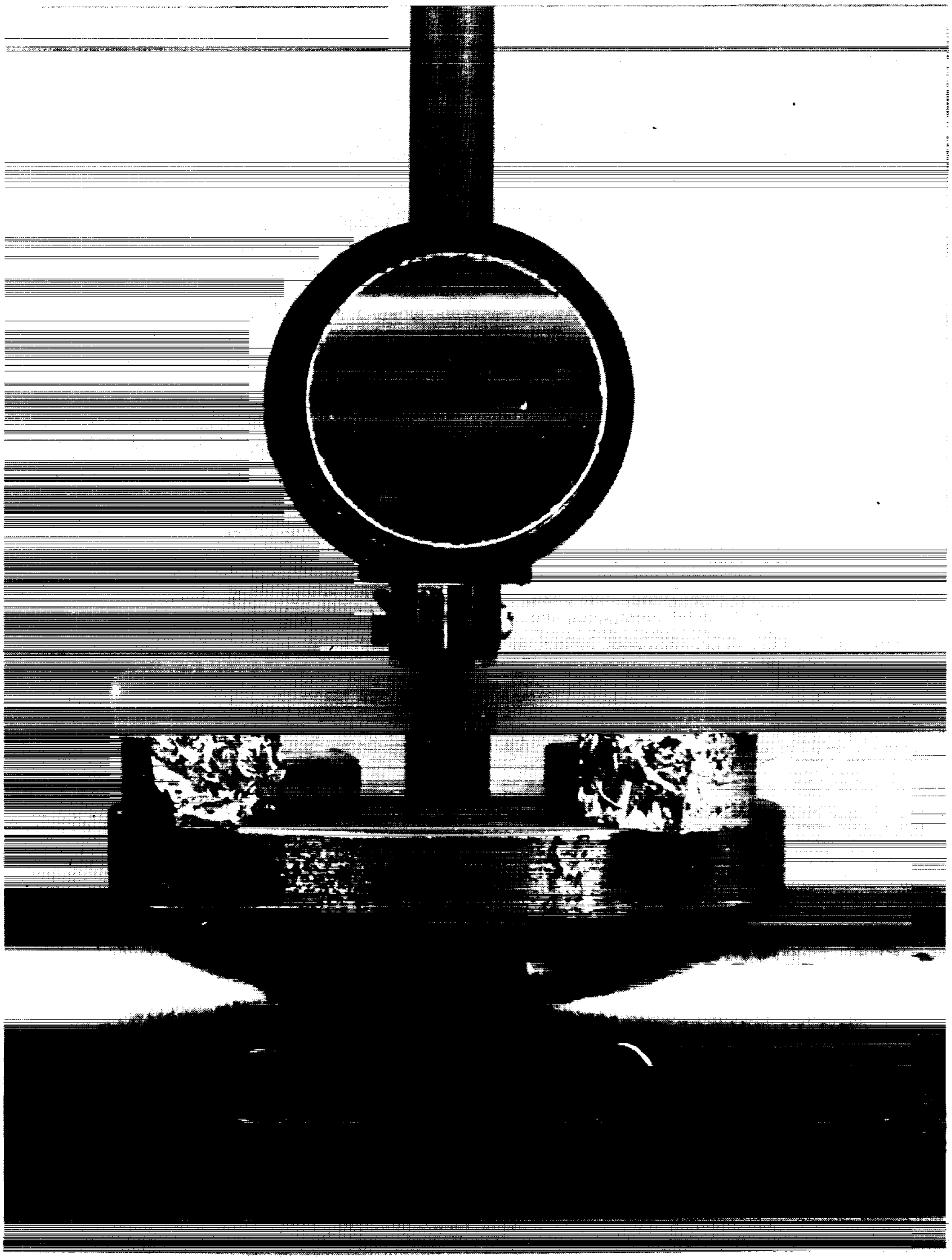

FIGURE 4. MAGNETICALLY MOUNTED MIRROR

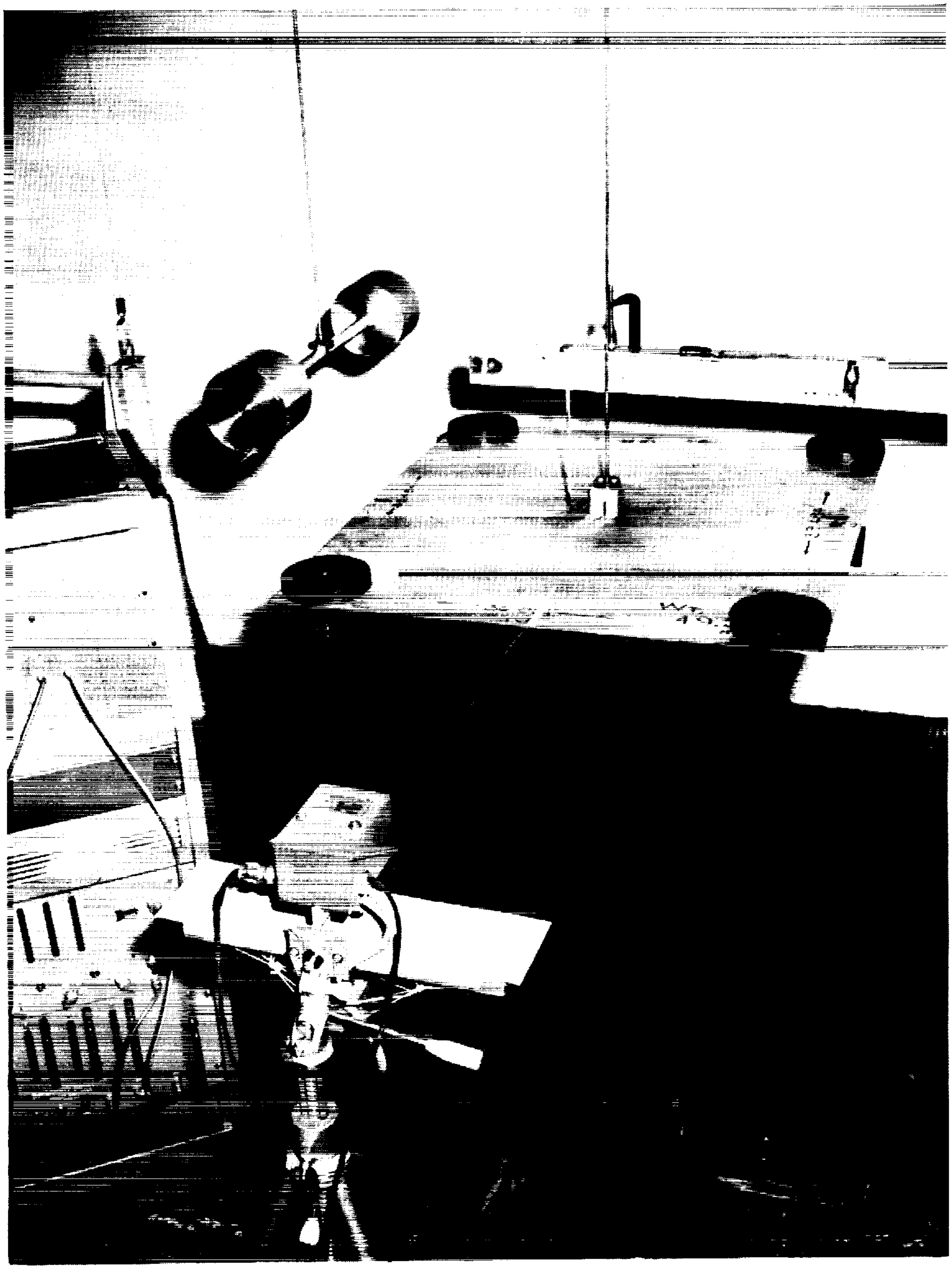

FIGURE 5. FLAT PLATE WITH ADDITIONAL INERTIA

position as to have relatively little external damping effect; (3) by rotating this object 90°, it can develop a very large external damping effect.

ANALYSIS OF METHOD

Inertia quantities determine a body's acceleration response to applied loads. If these loads produce a moment about a body axis, the body's inertia about this axis will resist this moment, so that the relationship $T = I\theta$ exists.

The theoretical angular twist for a torsion rod is $\theta = TL/GJ$. By setting $\theta = 1$ radian and solving for T, we have: $T = GJ/L$, which by definition is the torque stiffness of the torsion rod (i.e., torque required to twist the rod one radian). The symbol used for this torsion rod constant is "k".

It is apparent that "k" cannot be accurately calculated since it is dependent upon dimensions and material characteristics that change with load and/or temperature. Also, the accuracy of measurement and/or calculation of these rod characteristics are subject to considerable doubt. Therefore, the establishment of "k" is done by using "standard" inertia bodies whose calculated quantities are definitely obtainable. This is accomplished by measured investigations of the theoretical relationship: $I = kt^2/4\pi^2$.

This theoretical relationship assumes that "k" remains constant, and that environmental and operational characteristics of the system do not affect the time "t". If the temperature could be held constant, and if the operational characteristics such as angular velocities, air densities, air currents, etc., could also be held constant, then at least a fixed quantity of "additional mass effect" would be present, and could be accounted for analytically.

However, most facilities do not have environmental control, and the practical solution to the problem must provide a simple means of accounting for all influence factors existing during a varying and partially unknown environment. This can be done by measuring the amplitudes of twist in radians for a consecutive series of cycles and plotting the amplitude versus cycle curve of the oscillating system. (See FIG. 6, 7, 8, 9, and 10.) A straight line joining any two amplitude points on the curve provides the information necessary to determine the average efficiency of the system as it was operated from the larger amplitude on to the smaller amplitude, and the intermediate point on the curve whose tangent has the same slope as this straight line is the time instant during the operation when this efficiency is applicable.

The next problem to the solution is determining the time of the cycle whose midamplitude is that defined by the point of tangency of the efficiency line on the amplitude versus cycle curve.

This is done by drawing a series of lines parallel to the tangent line, reading the amplitudes and cycles at the two intersection points of these lines with the decay curve, and timing a series of runs using the starting amplitudes from these intersections and their corresponding number of cycles. The effective system torsional constant "C" can then be determined by solving the equation $C = I/Et^2$; this constant, having been determined by using standard inertia objects, can then be used as a system constant in subsequent measurements.

A sample work sheet is shown in Table 1.

Several sets of data and their associated calculations are presented in Appendix A for clarification of details and operating procedures.

Appendix B includes a detailed derivation of the static "k" or torque stiffness of the torsion rod, for comparison purposes.

An analytical analysis of the torsional system incorporating the efficiency factor "E" is explained in Appendix C.

CONCLUSIONS

A series of experimental investigations were conducted to devise a simple and accurate method for measuring mass moments of inertia. The method presented utilizes a torsion pendulum system and a selected group of "standard inertia bodies". The method developed is adaptable to any mechanical inertia measuring system.

TABLE 1. LARGE DUMBBELL

Decay Cycles	Radians	Temp. °C	Decay Cycles	Radians	Temp. °C	Decay Cycles	Radians	Temp. °C
0	0.3015	22.2	175	0.0856		410	0.0418	
5	0.2821		180	0.0842	22.8	420	0.0409	21.8
10	0.2636		185	0.0824		430	0.0400	
15	0.2465		190	0.0812		440	0.0388	21.7
20	0.2316	23.3	195	0.0796		450	0.0379	
25	0.2182		200	0.0782	23.0	460	0.0370	22.0
30	0.2061		205	0.0765		470	0.0363	
35	0.1954		210	0.0752		480	0.0354	22.6
40	0.1855	22.9	215	0.0741		490	0.0348	
45	0.1771		220	0.0728	23.2	500	0.0340	22.9
50	0.1692		225	0.0715		510	0.0332	
55	0.1621		230	0.0703		520	0.0325	23.2
60	0.1560	22.4	235	0.0692		530	0.0318	
65	0.1501		240	0.0682	23.5	540	0.0310	23.3
70	0.1448		245	0.0670		550	0.0305	
75	0.1398		250	0.0658		560	0.0300	23.6
80	0.1354	22.0	255	0.0647		570	0.0293	
85	0.1313		260	0.0637	23.8	580	0.0286	23.9
90	0.1274		270	0.0619		590	0.0280	
95	0.1240		275	0.0609		600	0.0275	23.8
100	0.1202	21.9	280	0.0600	23.9	650	0.0250	22.8
105	0.1169		285	0.0592		700	0.0226	22.0
110	0.1142		290	0.0580		750	0.0209	21.6
115	0.1112		295	0.0572		800	0.0191	22.7
120	0.1085	21.7	300	0.0563	23.8	850	0.0176	23.2
125	0.1062		310	0.0549		900	0.0162	23.9
130	0.1037		320	0.0533	23.2	950	0.0148	23.0
135	0.1012		330	0.0519		1000	0.0138	22.4
140	0.0992	22.0	340	0.0504	23.0	1050	0.0130	21.8
145	0.0968		350	0.0490		1100	0.0123	21.9
150	0.0951		360	0.0479	22.5	1150	0.0115	23.0
155	0.0930		370	0.0466		1200	0.0108	23.5
160	0.0912	22.2	380	0.0452	22.2	1250	0.0101	23.4
165	0.0891		390	0.0440		1300	0.00957	22.5
170	0.0873		400	0.0430	22.0	1350	0.00925	
						1400	0.0087	

Timer A 10,394.55006
Timer B 10,394.55410

The outlined results are indicative of the simplicity and accuracy that can be expected from a mechanical oscillating system. These results are summarized as follows:

1. In the theoretical analysis of mechanical oscillating systems, ideal conditions are assumed. In actuality, damping forces exist which alter theoretically expected results.

2. Analysis of decay rate curves indicates that damping is independent of inertia, but is directly influenced by environment, system configuration and friction (external and internal).

3. The tangent segment of the decay rate curve can be used to obtain an accurate system efficiency.

4. This system efficiency (or correction factor), when applied to the theoretical formula, will account for the effects of all damping forces. For the system outlined in this report, the following relationships exist:

Theoretically $I = kt^2/4\pi^2$

Actually $I = Ekt^2/4\pi^2$

(Note: A similar relation exists for all mechanical oscillating systems.)

5. Torsion rods can be proportioned to control oscillating frequency.

6. Results indicate that good accuracy can be obtained in an expedient manner with simple equipment.

APPENDIX A

The following is a series of examinations produced and analyzed by previously mentioned methods. Resulting data are noted comparatively with actual calculated values of these test specimens as conclusive evidence to support this theory.

For three test runs, the torsional system began at an amplitude of 0.20 radians for 100, 200, and 300 cycles. The slopes and tangent points (FIG. 11; Note: arrows denote tangents) were used to calculate the related efficiency factor "E". Times and temperatures were recorded and the mass moment of inertia determined.

Three other test runs were made in such a manner that the efficiency remained constant. Times were recorded and "Et^2" products compared.

Test Run	Radians	Cycles	Timer "A"	Timer "B"	Temperature in Centigrade
1.	0.20	100	741.94632	741.95363	23.3°
2.	0.20	200	1483.13212	1483.14957	23.0°
3.	0.20	300	2223.98744	2224.01825	22.3°
4.	0.20	410	3039.02179	3039.05071	22.9°
5.	0.15	260	1927.29040	1927.30544	22.5°
6.	0.10	76	563.39381	563.39590	21.9°

EFFICIENCY CALCULATIONS

$$E = \frac{2N\theta_M - \theta_1 + \theta_N}{2N\theta_M + \theta_1 - \theta_N}$$

$$E_1 = \frac{200(.135) - .20 + .0932}{200(.135) + .20 - .0932} = .9921$$

$$E_2 = \frac{400(.112) - .20 + .058}{400(.112) + .20 - .058} = .9937$$

$$E_3 = \frac{600(.094) - .20 + .040}{600(.094) + .20 - .040} = .9943$$

$$E_4 = \frac{820(.0839) - .20 + .029}{820(.0839) + .20 - .029} = .9950$$

$$E_5 = \frac{520(.0839) - .15 + .0416}{520(.0839) + .15 - .0416} = .9950$$

$$E_6 = \frac{152(.0839) - .10 + .0676}{152(.0839) + .10 - .0676} = .9949$$

The system constant (C) used in these calculations was calibrated at a temperature of 22.5°C.

Measured Mass Moment of Inertia Calculations
(Using Equation $I_O = ECt^2$)

Test Run	Answer in # in. sec^2
1.	I_O = .9921(1.75217)55.04898 = 95.6932
2.	I_O = .9937(1.75217)54.99261 = 95.7494
3.	I_O = .9943(1.75217)54.95760 = 95.7267
4.	I_O = .9950(1.75217)54.94189 = 95.7862
5.	I_O = .9950(1.75217)54.94782 = 95.7965
6.	I_O = .9949(1.75217)54.95390 = 95.7975

The actual mass moment of inertia of this "large dumbbell" as determined by calculation from the gaged physical dimensions is 95.772 # in. sec^2.

These results indicate that better accuracies are obtained by using a greater number of cycles in the "test run."

These differences may be attributed to one or more of the following: (1) gaging accuracies on physical dimensions used in calculations; (2) shift in tolerances of measuring equipment and "gear" hardware; (3) inconsistency of environment during actual tests; and (4) the plotting of the decay curve and the calculation of the efficiency therefrom.

Variables which may be neglected as applied individually but have an accumulative significance to very critical requirements are: (1) mass and inertia of the wand used to break the light source in the photoelectric counter and timing device; (2) drift of the mirror during a test; and (3) the resisting moment of inertia of the torsion rod.

APPENDIX B

A calibration setup was made to determine the static "k" (torque stiffness) of the torsion rod for comparison purposes.

The gear utilized for this examination was the "radian" protractor previously mentioned with the pointer light source and a system of pulleys and weights. Small ball bearings were used as pulleys to minimize friction. Two pieces of light weight nylon string were attached tangent to the periphery of a cylindrical mass 180° apart and equidistant from the rotational axis of the torsion rod. The free ends of these strings were passed through the pulleys in such a manner that weights could be fastened to apply a torque to the rod.

Weights were added to increase the torque in 0.0129-pound increments. The angle of twist, temperature, and tension load on the rod were recorded until the entire measuring range was covered. It was noted that the "k" would vary with temperature and tension load. A graph of "k" versus these individual variables was plotted. These curves appeared linear over a reasonable range and were extrapolated to determine the static torque stiffness of the torsion for 0° centigrade with no tension loading. These unstable conditions have a minute effect in practical application of the system, and may be neglected by keeping the "k" constant without any significant change in the end result. Values calculated from these data varied from the dynamic calibration by 0.5%, which may be attributed to the resisting effects of the string and pulleys.

APPENDIX C

The following is an analytical analysis of the torsional system incorporating the efficiency factor "E."

At the time of the rod constant "k" calibration, "k" appeared to be all that was necessary to obtain the degree of accuracy needed to satisfy the tolerances required for measured inertia. However, closer examination from this series of experiments revealed that, while the rod constant "k" was perfectly valid for a test dumbbell, it was not necessarily valid for other configurations. Since variables other than temperature are ever-present, i.e., aerodynamic forces, surface (or skin) friction, internal rod friction, etc., it becomes apparent that the mathematical formula $I_o = kt^2/4\pi^2$ must be altered or refined in such a manner that all existing variables are accounted for. Hence, the introduction of the efficiency factor "E."

It was noted that the additional mass effect has a direct relationship with the decay amplitudes, i.e., as the proportional mass effect is increased, the rate of decay increases. By plotting the decay amplitudes versus number of cycles, it is noted that at a large amplitude of swing, the rate of decay is high. (By the same token, the total efficiency of the system is low.) However, at a small amplitude the curve becomes near linear, making the decay rate low and the total efficiency high. Thus the rate of decay is inversely related to the total efficiency of the system. It is logical to assume that an efficiency factor "E" can be computed from that portion of the decay rate curve used to measure the mass moment of inertia.

The analytical aspects of the torsion rod inertia measuring system may be considered by fixing one end of a uniform rod of circular cross-section to a rigid support, and attaching a disk of homogeneous material with an inertia much greater than that of the rod to the lower end. The disk is fastened so that its cylindrical centerline coincides with the longitudinal axis of the rod.

Potential energy is then stored in the system by rotating the disk about its cylindrical axis through an angle θ. With the release of energy, the disk starts oscillating about its equilibrium position where $\theta = 0$. Referring to Figure 12, the applied torque or twisting moment is "fa."

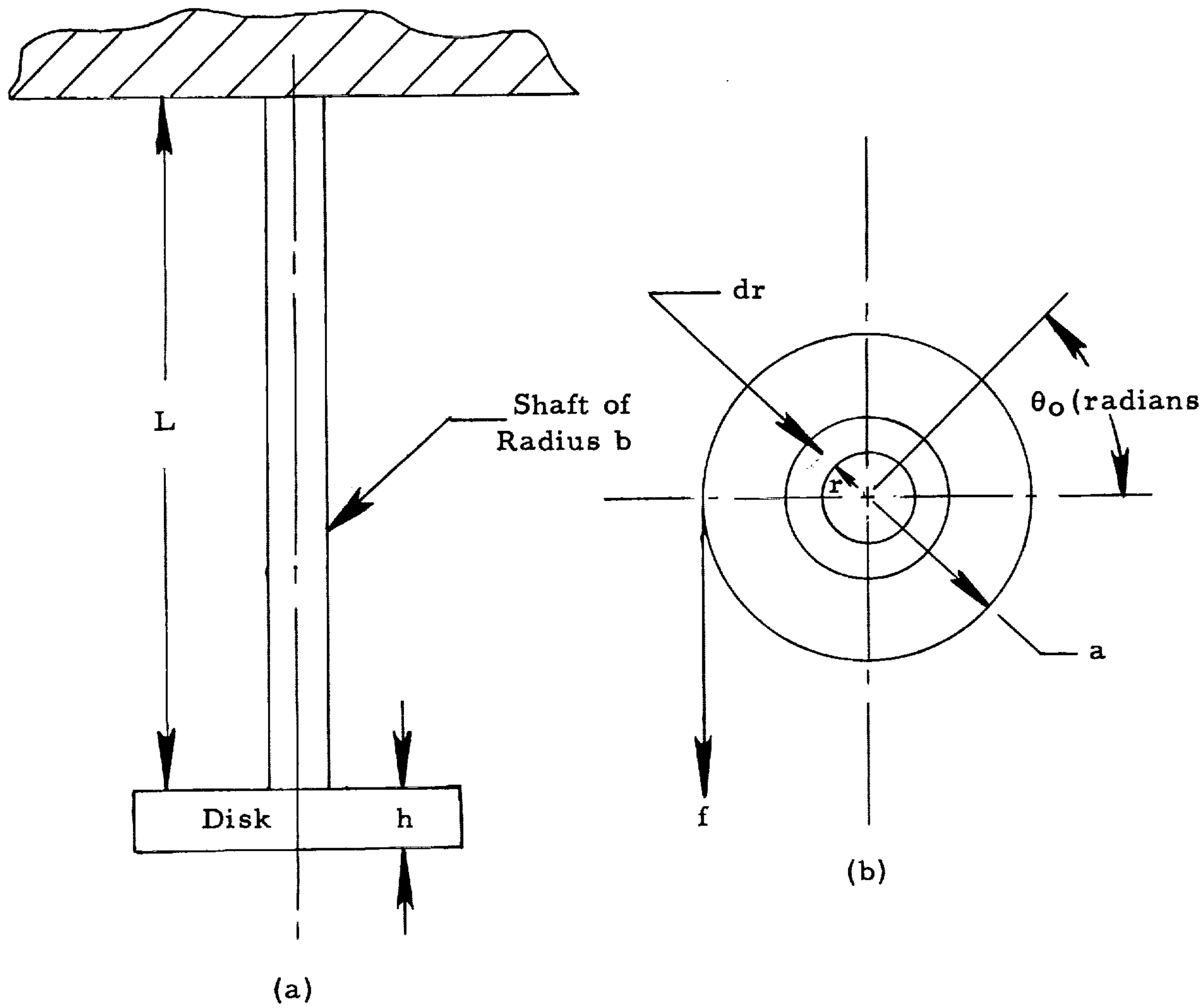

FIGURE 12. TYPICAL TORSIONAL SYSTEM

If the torque required to rotate the disk through an angle θ is "T," for a limited range of θ, a linear relationship $T = k\theta$ exists. When the elastic limit of the torsion rod is not exceeded, the torque-stiffness constant of the rod "k" is defined as the twisting moment required to cause an angular movement of 1 radian. The torque compliance is defined as "k^{-1}"; for a uniform shaft, $k = \pi b^4 G/2L = GJ/L$.

The condition to be satisfied during oscillation (after removal of "f") is that the algebraic sum of the internal torques shall be zero. The torque due to the resisting moment for the shaft is given by

$T = k\theta$, and neglecting the inertia of the shaft (this was considered in the experiment, but was found to be insignificant for practical use), there remains the inertial torque due to the disk. For example, referring to Figure 12b, the mass of an elemental ring is 2 phr dr. The peripherial velocity of the ring is $d(r\theta)dt = rd\theta/dt$, and its moment of momentum $(2\pi phr\ dr)\ r^2\dot{\theta} = (2\theta phr^3dr)\dot{\theta}$. The time rate of change of the moment of momentum of the ring is equal to the inertial torque, so: $dT_1 = (2\pi phr^3dr)\ddot{\theta}$. For the whole disk:

$T_1 = 2\pi ph\ddot{\theta} \int_0^a r^3dr = (p\pi a^4h/2)\ddot{\theta} = I\ddot{\theta}$, where: $I = p\pi a^4h/2$ $= Ma^2/2$, is the moment of inertia of the disk.

Then, by $T = k\theta$, and $T_1 = I\ddot{\theta}$, the above condition is satisfied if: $I\ddot{\theta} + t\theta = 0$, or $\ddot{\theta} + w_1^2\theta = 0$, where $w_1^2 = k/I$. Its complete solution is: $\theta = A\cos w_1t + B\sin w_1t$, and the angular frequency about the common axis of the torsion rod and the disk is $w_1 = (2k/p\pi a^4h)^{1/2} = (b/a)^2(G/ph)^{1/2}/L^{1/2}$; thus the size of a torsion rod may be mathematically selected to meet specific requirements.

The theoretical period of oscillation is $t = 2\pi(I_O/k)^{1/2}$, I_O being the mass moment of inertia of the entire body attached to the "free" end of the torsion rod; therefore, the theoretical mass moment of inertia of a given body or test object is: $I_O = kt^2/4\pi^2$, or $I_O = Ct^2$ where C is the system constant $(k/4\pi^2)$.

Actually, there are both internal and external forces that cause successive decays to the theoretical amplitude of an oscillating system. Some of these forces cause "coulomb" damping which results in a constant rate of decay with respect to amplitude, but does not affect the time per cycle. Other forces cause "viscuous" damping which results in a varying rate of amplitude decay and a varying rate of change of time per cycle.

The combined effort of these two groups of resisting forces can be accounted for by a proper interpretation of the amplitude decay curve combined with the measured time per cycle of that part of the decay curve used.

The resulting correction required to reduce experimental results to theoretical true values is often called "Additional Mass Effect" and may be visualized as an increased radius of gyration.

In order to obtain near exact results in measuring mass moments of inertia, a correction factor to account for these damping forces must be incorporated in the mathematical equation used. An analysis of decay curves (radians of twist versus number of cycles) from a series of tests provided a method of establishing such a correction factor for any one, any consecutive group, and/or for the complete series of energy twists generated.

The slope of a straight line joining the starting and ending points of this curve (chord line) determines the efficiency for the series of energy twists within the intercepts of the curve and chord. The amplitude used in the efficiency formula is the amplitude corresponding to the point of tangency that a line parallel to the chord line makes with the curve. The "E" (efficiency factor) should be computed correspondingly:

1. The energy used during any series of energy twists $= C(\theta_1 - \theta_N)$.

2. The average energy used during one energy twist $= C\left(\frac{\theta_1 - \theta_N}{2N}\right)$.

3. The starting energy for that one cycle having the same efficiency as the entire test run $= C\left[\theta_M + \left(\frac{\theta_1 - \theta_N}{2N}\right)\right]$ or "input."

4. The corresponding ending energy for this one cycle $= C\left[\theta_M - \left(\frac{\theta_1 - \theta_N}{2N}\right)\right]$ or "output."

5. For any mechanical system: efficiency = input/output; therefore, for any mechanical oscillating system:

$$E = C\left[\theta_M - \left(\frac{\theta_1 - \theta_N}{2N}\right)\right] \div C\left[\theta_M + \left(\frac{\theta_1 - \theta_N}{2N}\right)\right] \quad \text{or}$$

$$E = \frac{2N\theta_M - \theta_1 + \theta_N}{2N\theta_M + \theta_1 - \theta_N} \quad \text{for the test run used.}$$

In the formula $I_o = ECt^2$, the "t" (average time for one cycle) should be computed by dividing the total time (in seconds) taken in a test run by the total number of cycles in that particular test run.

SELECTED BIBLIOGRAPHY

N. W. Melachlan, D. Sc.: Theory of Vibrations, Dover Publications, Inc.

Baumann, Robert C.: Vanguard Satellite Spin-Reduction Mechanism, NASA TN D-496.

Simkovich, A., and Baumann, Robert C.: Balancing Vanguard Satellite, NASA TN D-498.

Bandeen, W. R., and Manger, W. P.: Angular Motion of the Spin Axis of the Tiros I Meteorological Satellite Due to Magnetic and Gravitational Torques, NASA TN D-571.

Eide, Donald G.: Experimental Evaluation of the Deceleration of Aluminum Cylinders Rotating in a Magnetic Field and Comments on Magnetic Damping of a Flywheel Control, NASA TN D-749.

Grantham, William D.: Effects of Mass - Loading Vibrations and Applied Moments on Motion and Control of a Manned Rotating Space Vehicle, NASA TN D-803.

Soule, Hartley A., and Miller, Marvel P.: The Experimental Determination of the Moments of Inertia of Airplanes, NACA Report No. 467.

Gracey, William: The Experimental Determination of the Moments of Inertia of Airplanes by a Simplified Compound-Pendulum Method, NACA TN No. 1629.

Turner, Howard L.: Measurement of the Moments of Inertia of an Airplane by a Simplified Method, NACA TN 2201.

Boucher, Robert W., Rich, Drexel A., Crane, Harold L., and Matheny, Cloyce E.: A Method for Measuring the Product of Inertia and the Inclination of the Principal Longitudinal Axis of Inertia of an Airplane, NACA TN 3084.

Cole, Henry A., Jr., and Bennion, Frances L.: Measurement of the Longitudinal Moment of Inertia of a Flexible Airplane, NACA TN 3870.

Williams, H. E.: On the Motion of a Slender Rigid Body Caused by a Small Torque, JPL TR 32-62.

Porter, John C., Jr.: Critical Speed of Rotation and Dynamic Moment per Unit Angle of Misalignment of the Loki, JPL Progress Report No. 20-221.

Skogh, Jorgen: A Unique Solution to the Moment of Inertia Problem, S. A. W. E. TP No. 214.

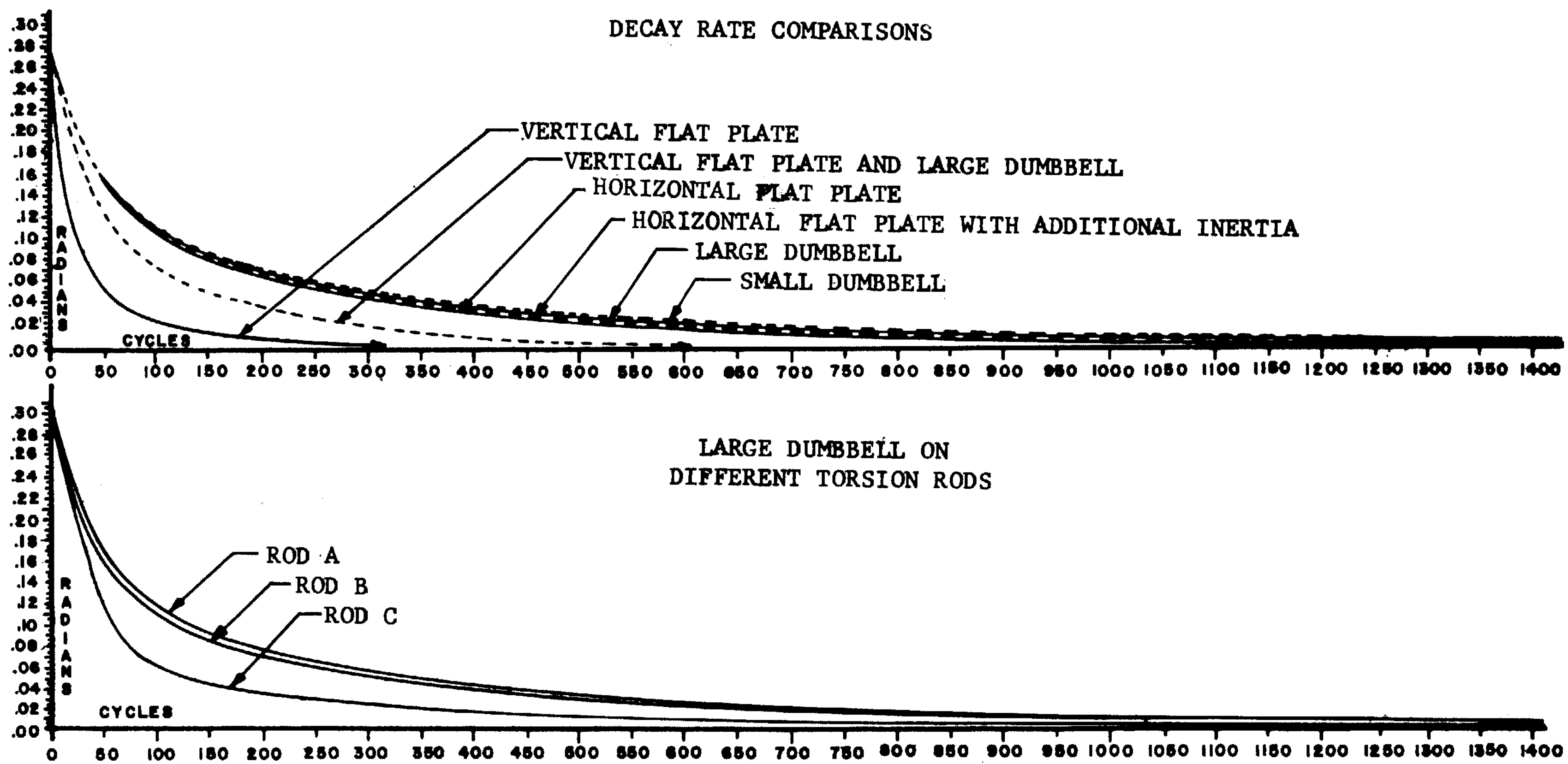

FIGURE 6. AMPLITUDE VERSUS CYCLE CURVE

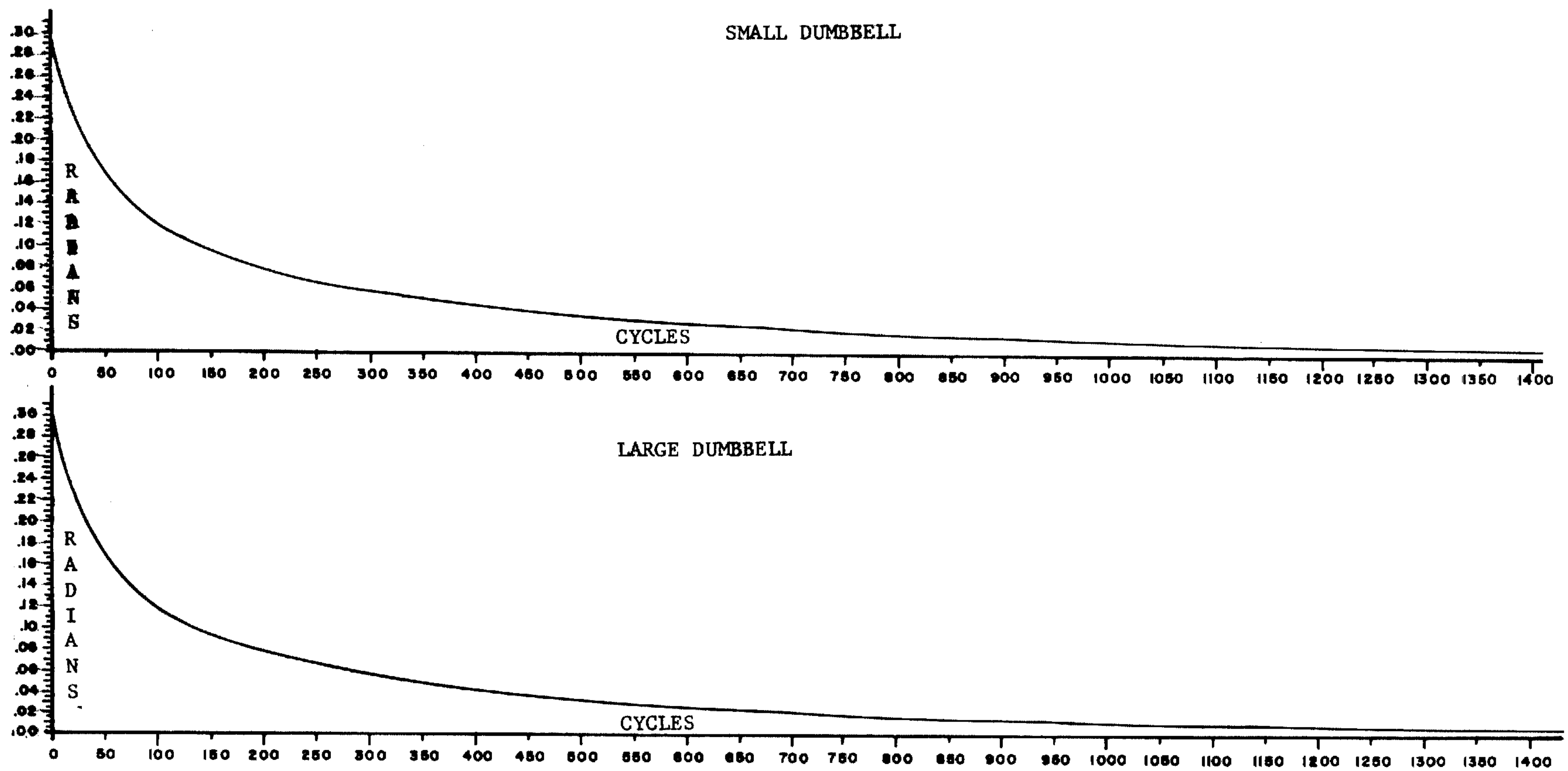

FIGURE 7. AMPLITUDE VERSUS CYCLE CURVE

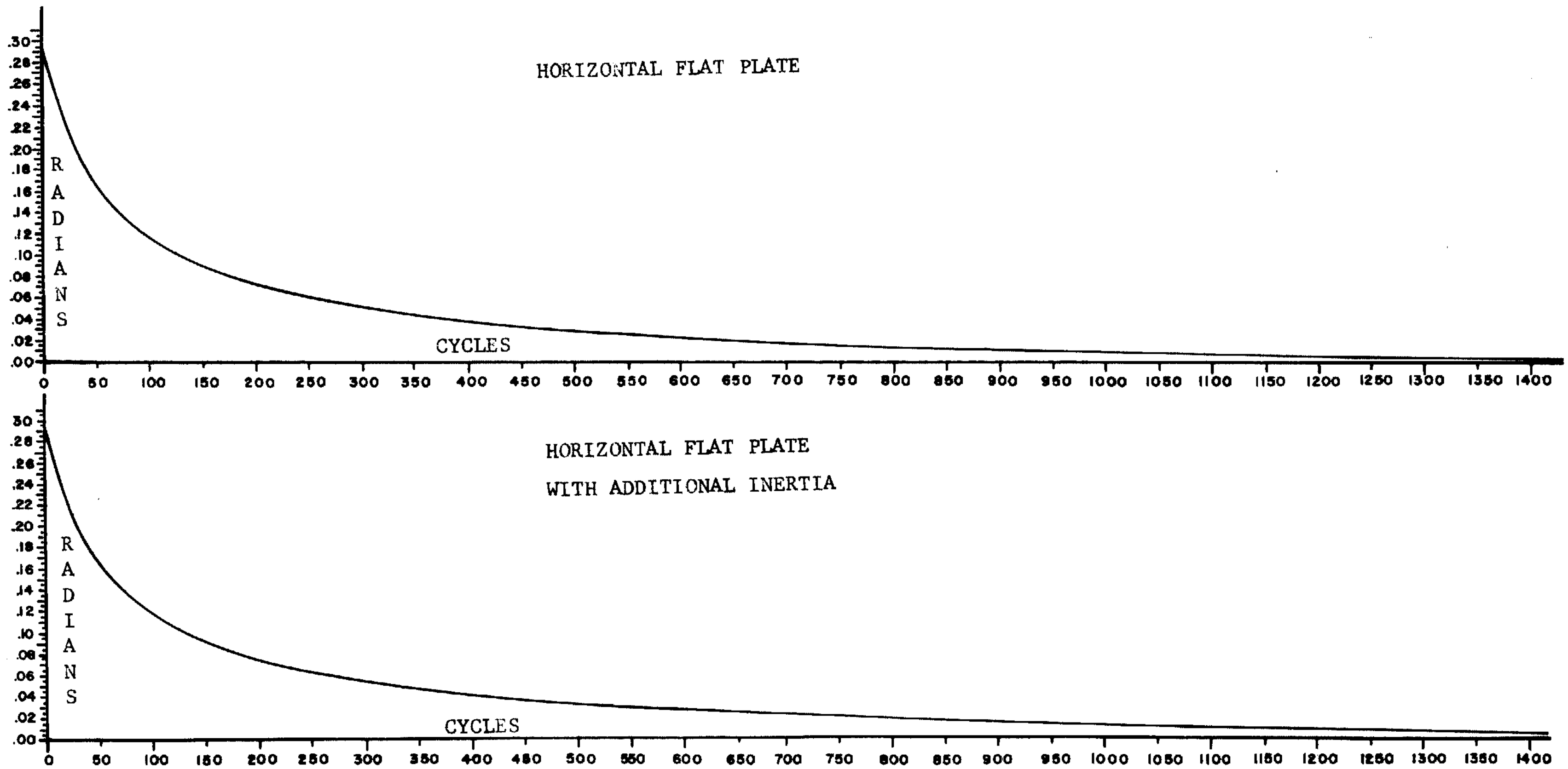

FIGURE 8. AMPLITUDE VERSUS CYCLE CURVE

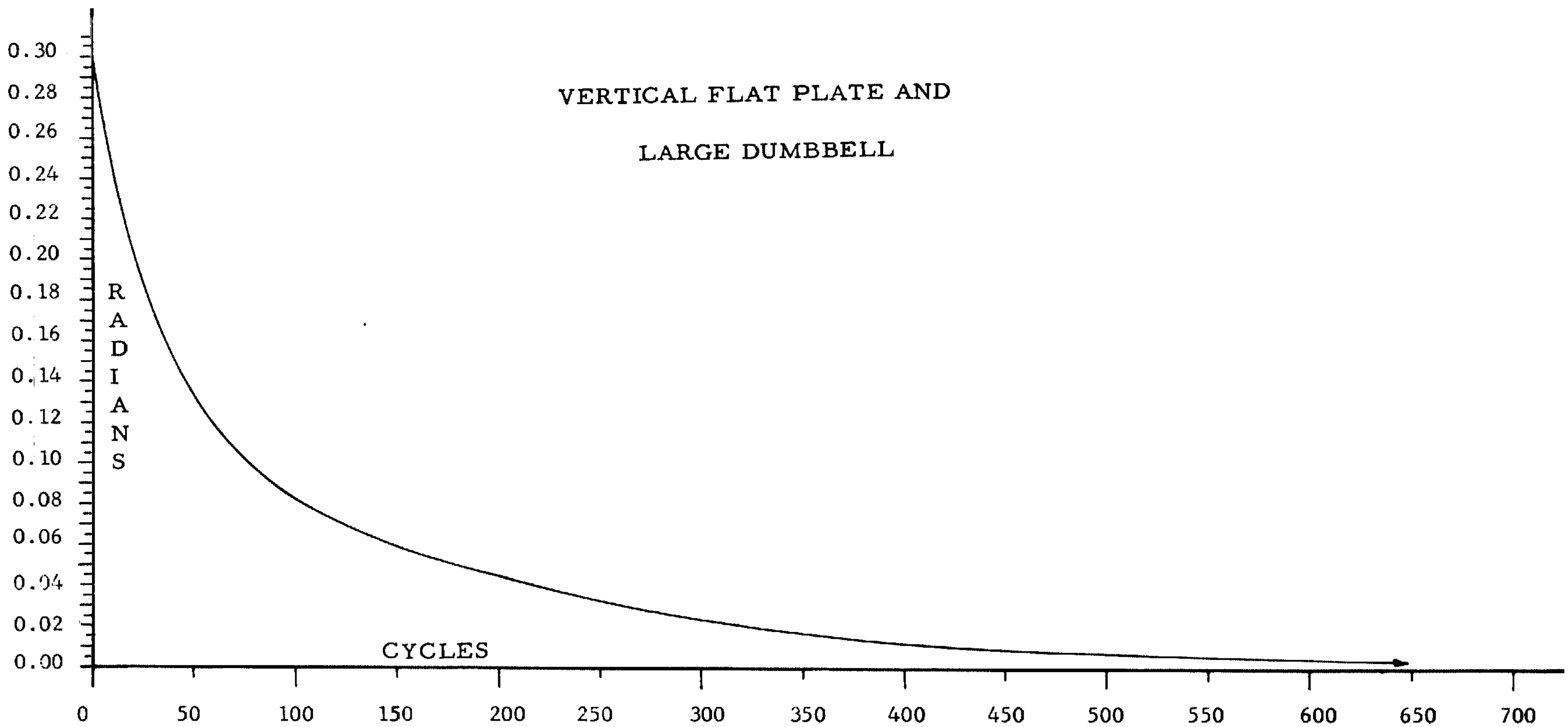

FIGURE 9. AMPLITUDE VERSUS CYCLE CURVE

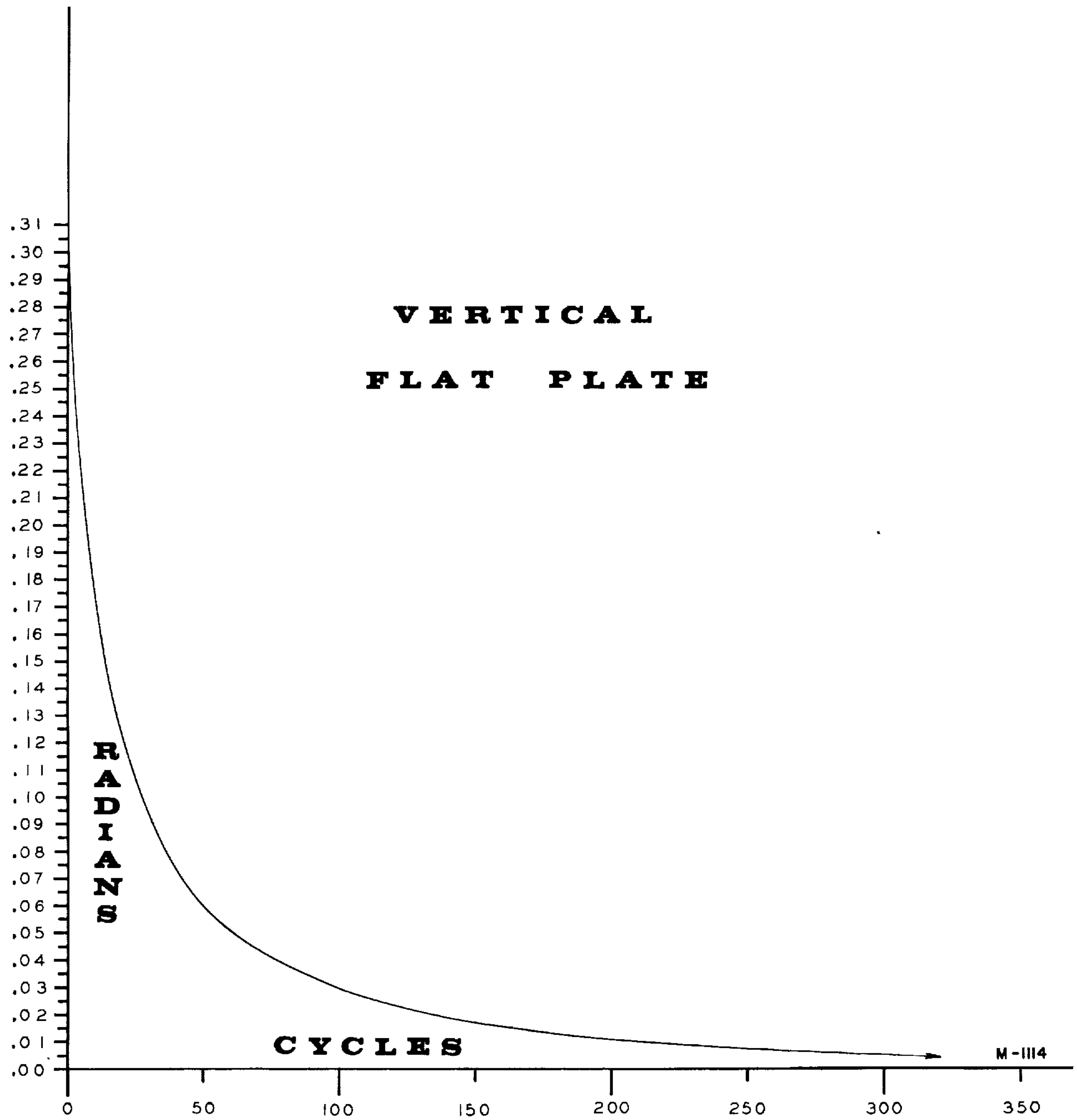

FIGURE 10. AMPLITUDE VERSUS CYCLE CURVE

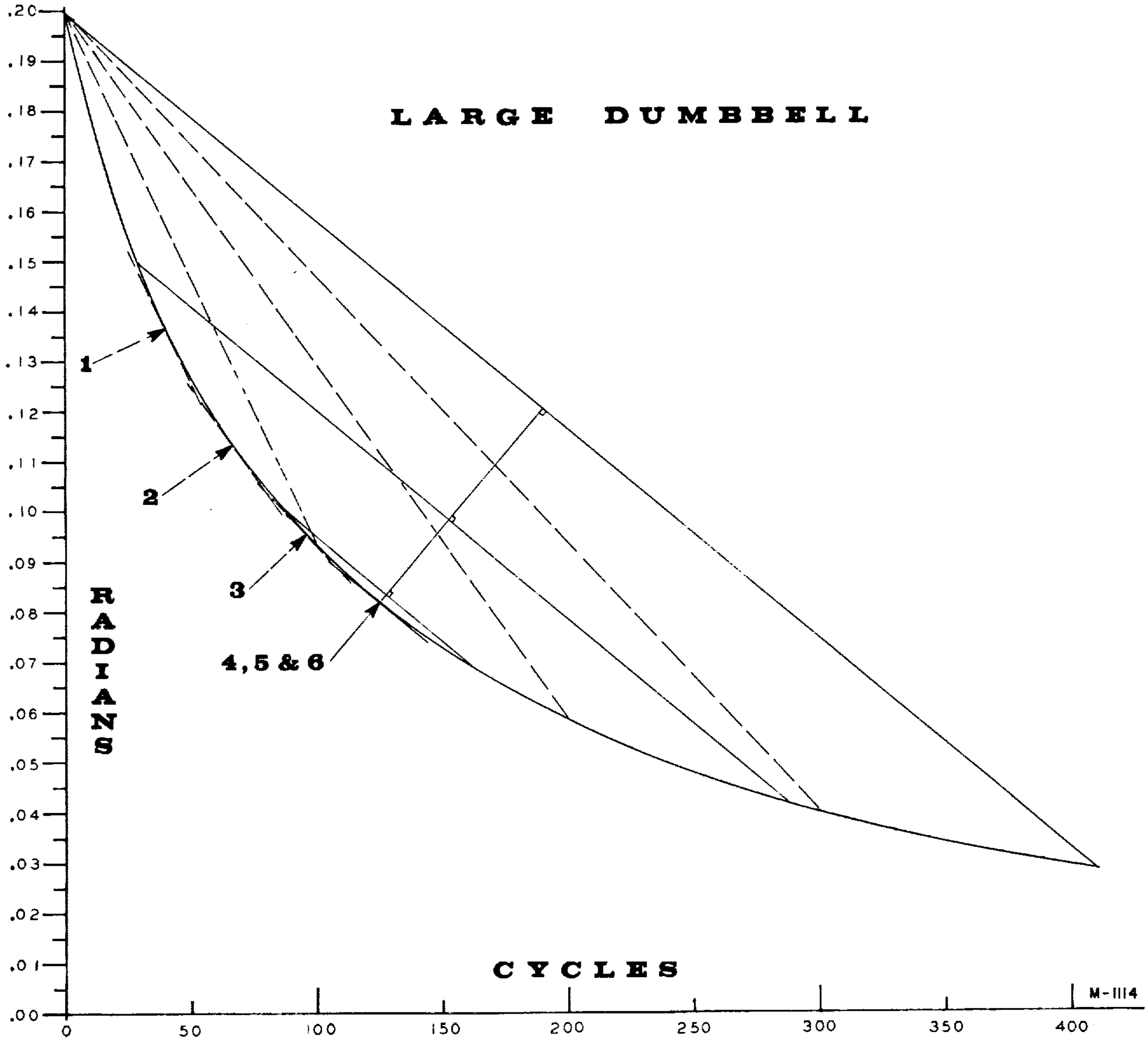

FIGURE 11. SAMPLE EFFICIENCY DERIVATION CURVE

NASA TN D-1114
National Aeronautics and Space Administration.
DETERMINING INERTIAS BY USING THE AMPLITUDE DECAY RATE OF A MECHANICAL OSCILLATING SYSTEM. Gene T. Carpenter and Dan T. Meredith. May 1962. 28p. OTS price, $0.75. (NASA TECHNICAL NOTE D-1114)

A series of tests were performed on a torsional system to develop a rapid and accurate method for measuring mass moments of inertia. Amplitude versus cycle curves from these tests provided a means of establishing the efficiency at which the torsional system functions. Investigations revealed that amplitude decay or damping is independent of mass and inertia for a specific object, but is directly related to some combination of environment and system configuration. This relationship is determined and applied to inertia measuring methods.

I. Carpenter, Gene T.
II. Meredith, Dan T.
III. NASA TN D-1114

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